Extraction of cluster parameters from Sunyaev-Zeldovich effect observations with simulated annealing optimization

Steen H. Hansen

University of Zurich, Winterthurerstrasse 190, 8057 Zurich, Switzerland

Abstract

We present a user-friendly tool for the analysis of data from Sunyaev-Zeldovich effect observations. The tool is based on the stochastic method of simulated annealing, and allows the extraction of the central values and error-bars of the 3 SZ parameters, Comptonization parameter, y, peculiar velocity, v_p, and electron temperature, T_e. The f77-code `SASZ` will allow any number of observing frequencies and spectral band shapes. As an example we consider the SZ parameters for the COMA cluster.

1 Introduction

Galaxy clusters typically have temperatures of the order keV, $T_e = 1 - 15$ keV, and a CMB photon which traverses the cluster and happens to Compton scatter off a hot electron will therefore get increased momentum. This up-scattering of CMB photons, which results in a small change in the intensity of the cosmic microwave background, is known as the Sunyaev-Zeldovich (SZ) effect, and was predicted just over 30 years ago (Sunyaev & Zeldovich 1972). The first radiometric observations came few years later (Gull & Northover 1976, Lake & Partridge 1977), and while recent years have seen an impressive improvement in observational techniques and sensitivity (Laroque et al. 2002, De Petris et al 2002), then the near future observations will see another boost in sensitivity by orders of magnitude. These include dedicated multi-frequency SZ observations like ACT [1] and SPT [2]. The SZ effect will thus soon provide us with an independent description of cluster properties, such as evolution and radial profiles. For recent excellent reviews see (Birkinshaw 1999, Carlstrom, Holder & White 2002).

[1] `http://www.hep.upenn.edu/~angelica/act/act.html`
[2] `http://astro.uchicago.edu/spt/`

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The SZ effect is traditionally separated into two components according to the origin of the scattering electrons

$$\begin{aligned}\frac{\Delta I(x)}{I_0} &= \Delta I_{\rm thermal}\,(x, y, T_e) + \Delta I_{\rm kinetic}\,(x, \tau, v_p, T_e) \\ &= y\,\left(g(x) + \delta_T(x, T_e)\right) - \beta\tau\,\left(h(x) + \tilde{\delta}_{kin}(x, T_e)\right)\,,\end{aligned} \tag{1}$$

with $x = h\nu/kT_{cmb}$ and $I_0 = 2(kT_{cmb})^3/(hc)^2$ where $T_{cmb} = 2.725$ K. The first term on the rhs of eq. (1) is the thermal distortion with the non-relativistic spectral shape

$$g(x) = \frac{x^4\,e^x}{(e^x - 1)^2}\left(x\,\frac{e^x + 1}{e^x - 1} - 4\right)\,, \tag{2}$$

and the magnitude is given by the Comptonization parameter

$$y = \frac{\sigma_{\rm T}}{m_e\,c^2}\int dl\, n_e\, kT_e\,, \tag{3}$$

where m_e and n_e are masses and number density of the electrons, and σ_T is the Thomson cross section.

For non-relativistic electrons one has $\delta_T(x, T_e) = 0$, but for hot clusters the relativistic electrons will slightly modify the thermal SZ effect (Wright 1979). These corrections are easily calculated (Rephaeli 1995, Itoh & Nozawa 2003, Ensslin & Kaiser 2000, Dolgov et al 2001), and can be used to measure the cluster temperature purely from SZ observations (Hansen, Pastor & Semikoz 2002). For the implementation below we will use an extension of the method developed in Aghanim et al. (2003), using a fit to the spectral shape of $\delta_T(x, T_e)$, which everywhere in the range $20 - 900$ GHz and $T_e < 24$ keV is very accurate, $|\delta_T^{fit} - \delta_T|/(|\delta_T| + |g|) < 0.005$. In the range $24\,\mathrm{keV} < T_e < 100$ keV the accuracy is slightly lower.

The kinetic distortions have the spectral shape

$$h(x) = \frac{x^4\,e^x}{(e^x - 1)^2}\,, \tag{4}$$

and the magnitude depends on $\beta = v_p/c$, the average line-of-sight streaming velocity of the thermal gas (positive if the gas is approaching the observer), and the Thomson optical depth

$$\tau = \sigma_{\rm T_e}\int dl\, n_e\,. \tag{5}$$

Thus, when the intra-cluster gas can be assumed isothermal one has $y = \tau k T_e/(mc^2)$. For large electron temperatures there are also small corrections to the kinematic effect, $\tilde{\delta}_{kin}(x, T_e) \neq 0$ (Sazonov & Sunyaev 1998, Nozawa, Itoh, & Kohyama 1998), an effect which is negligible with present day sensitivity.

Given the different spectral signatures of $g(x), h(x)$ and $\delta_T(x, T_e)$, it is straight forward to separate the physical variables y, v_p and T_e from sensitive multi-frequency observation. However, due to the complexity of the spectral shapes, in particular of $\delta_T(x, T_e)$, the parameter space spanned by y, v_p and T_e may be non-trivial with multiple local minima in χ^2. We therefore present a stochastic analysis tool `SASZ` based on simulated annealing, which allows a safe and fast parameter extraction even for such a complex parameter space.

It is worth noting that whereas we here choose to use the set of cluster variables, (y, T_e, v_p), which are easily understood physically, then the analysis could be simplified significantly by introducing a set of *normal* parameters whose likelihood function is well-approximated by a normal distribution (Kosowsky, Milosavljevic & Jimenez 2002, Chu, Kaplinghat & Knox 2003). This set of normal parameters could e.g. be $(y, T_e, K = \tau v_p)$, which directly enter in eq. (1). Another set could be $(y, T_e, \tilde{K} = v_p T_e^{-0.85})$, where $\tilde{K}$ enters because the cross-over frequency, ν_0, is easily determined observationally (due to a fast variation of $\Delta I(x)$) and the fact that this cross-over frequency to a good approximation depends only on T_e and $\tilde{K}$

$$\nu_0 = 217.4\,(1 + 0.0114\,T_5) + 12\,T_5^{-0.85}\,v_{500}\,\mathrm{GHz}\,, \tag{6}$$

using $T_5 = T_e/(5\mathrm{keV})$ and $v_{500} = v_p/(500\mathrm{km/sec})$.

2 Simulated annealing

Let us now discuss the stochastic method used in `SASZ` . The idea behind the technique of simulated annealing is as follows. If a thermodynamic system is cooled down sufficiently slowly then thermodynamic equilibrium will be maintained during the cooling phase. When the temperature approaches zero, $T_A \to 0$, then the lowest energy state of the system, E_{min}, will be reached (Kirkpatrick, Gelatt, & Vecchi 1983).

One can use this idea to search for the minimum in the space of allowed parameters, in which case the energy is replaced with $E = \chi^2$. The method is related to Monte Carlo methods, because one is basically jumping randomly around in parameter space, and if a given new point has lower energy (that is lower χ_i^2 in our case) than the previous point (χ_{i-1}^2), then this point is accepted. To ensure that a system is not trapped in a local minimum there is

a certain probability of keeping the new point even if its energy may be larger than the previous one (Metropolis et al. 1953)

$$P_{\text{accept}} = \begin{cases} 1 & \text{if} \quad E_i \leq E_{i-1} \\ e^{-(E_i - E_{i-1})/T_A} & \text{if} \quad E_i > E_{i-1} \end{cases}$$

and this probability, P_{accept}, will then be compared to a random number between 0 and 1, to decide if the point will be kept or not. The temperature of the system, T_A, is then slowly lowered to ensure that the global minimum χ^2 is reached. We emphasize that the simulated annealing temperature, T_A, is completely unrelated to the electron temperature, T_e, of the galaxy cluster.

In reality the jumping in parameter space is not completely random, instead the new points are drawn according to

$$x_i = x_{i-1} + A_\beta \sqrt{\frac{T_A}{T_0}}\, ran \tag{7}$$

where ran is a random number between $-1/2$ and $1/2$. The size of the jump is such that initially all of parameter space is easily sampled, $A_\beta \approx (x_{\max} - x_{\min})$, whereas for a cooled system only very small jumps are allowed due to the $\sqrt{T_A}$ in eq. (7). For flat directions in parameter space (such as T_e and v_p) the coefficient in A_β is a factor 10 larger than for the dominating Comptonization parameter y. In the first version of `SASZ` x is a 3 dimensional vector, $\vec{x} = (y, T_e, v_p)$, as energy we are using $E = \chi^2$, and we allow approximately 1000 random jumps at each temperature step. The cooling scheme is adaptive according to how many point are accepted or rejected, but can trivially be fixed to an exponential cooling scheme, $T_A(j) = c\,T_A(j-1)$, where $c < 1$. We use $T_0 = 1$ and $T_{\text{final}} = 10^{-12}$, and the parameter space allowed is presented in Table 1.

Parameter	Allowed range		Units
$log(y)$	-7	-2	
T_e	0	100	keV
v_p	-10^4	10^4	km/sec

We note that the technique of simulated annealing, which is often used for problems like the travelling salesman problem, has previously been used is cosmology, e.g. for parameter extraction from cosmic microwave background radiation data (Knox 1995, Hannestad 2000).

2.1 How to use `SASZ` *in practice?*

The user has a set of observing frequencies (and possibly frequency bands) with corresponding SZ observations. The user specifies if some of the parameters have been observed with other methods; the temperature can be known to be e.g. $T_e = 12 \pm 1$ keV, or the peculiar velocity can be known e.g. $v_p = 300 \pm 500$ km/sec. The Fortran77 code `SASZ` will then stochastically analyse the parameter space (as described above) and presents the central value and 1σ error-bars for the cluster parameters (y, v_p, T_e). The method is much faster than e.g. the method of automatic refining grids (Aghanim et al. 2003). A user guide with examples can be downloaded together with the code [3]. This user guide also explains how to implement measured frequency band if available.

2.2 Error-bars

There are two possible steps for determination of error-bars. The first is a very fast Gaussian approximation, and the second is a more accurate method which simultaneously provides data for nice figures.

The Gaussian estimate of the error-bars comes for free. While searching for the global minimum `SASZ` will remember some intermediate points, $\vec{x}_i$, and their χ_i^2. For each of the parameters, e.g. T_e, one can consider the 2-dimensional figure $(T_e, \Delta\chi^2)$, where all points lie within a quadratic curve. One can fit a quadratic curve through the best fit point and any of the other points. This method gives a fairly good estimate of the error-bar by performing such fit for all the points and then choosing the largest value. If upper and lower error-bars are different from each other, then the bigger is chosen for simplicity. The code selects a certain number of point for the fit, and presents automatically the error-bars for the 3 SZ parameters.

The second method is to make a grid in the parameter we are interested in, and then minimize over the other parameters for each point on that grid. In this way one can get different upper and lower error-bars more accurately. This method is, however, slower because one will have to run another optimization for each chosen point in the grid. `SASZ` automatically volunteers to perform such a calculation in the 2σ range found with the Gaussian method or within a user-defined range.

[3] `http://krone.physik.unizh.ch/~hansen/sz/`

3 An example: COMA

As an example of the use of SASZ we consider the nearby cluster COMA at redshift, $z = 0.0231 \pm 0.0017$. This cluster has been measured at 6 different frequencies, 32 GHz (Herbig et al 1995), 143, 214 and 272 GHz (De Petris et al 2002), and at 61 and 94 GHz (Bennett et al. 2003). For a thorough discussion of consistency and a combined analysis see Battistelli et al. (2003), where the results are combined in their Table 1.

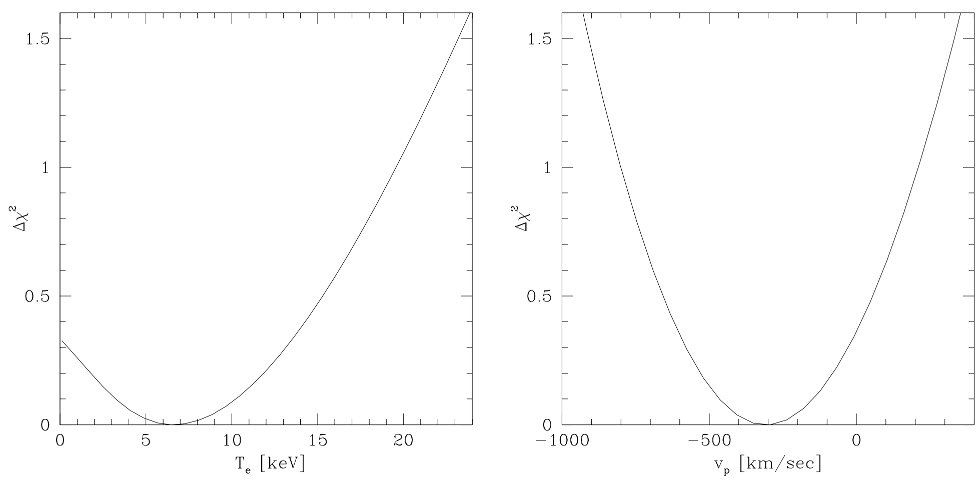

Fig. 1. Using SASZ to determine the electron temperature or peculiar velocity for COMA. Figure 1a shows $\Delta\chi^2$ as a function of the electron temperature, T_e. Other parameters have been maximized over. The 1σ error-bar on T_e (corresponding to $\Delta\chi^2 = 1$) gives $T_e < 20$ keV. Figure 1b shows $\Delta\chi^2$ as a function of the peculiar velocity, v_p. Other parameters have been maximized over. The 1σ error-bar on v_p (corresponding to $\Delta\chi^2 = 1$) gives $v_p = -300 \pm 500$ km/sec.

Let us first consider the effect of bandwidth on the determination of y. We look at the case where the temperature is measured, $T_e = 8.25 \pm 0.10$ keV (Arnaud et al. 2001), and the peculiar velocity is known, $v_p = -29 \pm 299$ km/sec (Colless et al 2001). We consider 3 cases, **a)** delta function for the spectral band shape, **b)** Gaussian shape, and **c)** flat (top-hat) shape. For the Gaussian and top-hat shapes we use the band widths from Table 1 in Battistelli et al. (2003). We find **a)** $log(y) = -4.081^{+0.047}_{-0.048}$, **b)** $log(y) = -4.080^{+0.039}_{-0.055}$, and **c)** $log(y) = -4.077^{+0.040}_{-0.054}$. We thus see, that the differences between the filters have almost no effect on the central value of the Comptonization parameter, and fairly small effect on the error-bars of $log(y)$. This is good news (and in agreement with the findings of Church, Knox, & White (2003)), because the use of filters makes the computation much longer since one effectively must calculate at many more frequencies. Leaving v_p as a free parameter to be maximized over has little effect, but leaving T_e as a completely free parameter instead will increase the error-bars on $log(y)$ by almost 50%.

Using Gaussian filters and assuming the peculiar velocity is measured, $v_p = -29 \pm 299$ km/sec (Colless et al 2001), one finds that COMA has a temperature of $T_e = 6.6 \pm 13$ keV, as seen on figure 1a. This is in good agreement with the X-ray determination, but still with much larger error-bars. In comparison, the first cluster temperature measurement using purely the SZ observations gave $T_e = 26^{+34}_{-19}$ keV for A2163 (Hansen, Pastor & Semikoz 2002), where only observations at 4 frequencies were available. Similarly, using the observed temperature, $T_e = 8.25 \pm 0.10$ keV (Arnaud et al. 2001), one finds the peculiar velocity $v_p = -300 \pm 500$ km/sec, as seen on figure 1b, which is in good agreement with the findings of Colless et al (2001).

4 Conclusion

We are presenting a user-friendly tool for parameter extraction from Sunyaev-Zeldovich effect observations. The tool `SASZ` is based on the stochastic method of simulated annealing, and is useful for any number of observing frequencies and any spectral band shape. The first version of the tool allows a determination of the Comptonization parameter, y, the peculiar velocity, v_p, and the electron temperature, T_e. The f77 code `SASZ` can be readily downloaded together with a user guide with examples from
`http://krone.physik.unizh.ch/~hansen/sz/` .

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